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On the role of the current loss in radio pulsar evolution

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Abstract The aim of this article is to draw attention to the importance of the electric current loss in the energy output of radio pulsars. We remind that even the losses attributed to the magneto-dipole radiation of a pulsar in vacuum can be written as a result of an Ampere force action of the electric currents flowing over the neutron star surface (Michel 1991; Beskin, Gurevich & Istomin 1993). It is this force that is responsible for the transfer of angular momentum of a neutron star to an outgoing magneto-dipole wave. If a pulsar is surrounded by plasma, and there is no longitudinal current in its magnetosphere, there is no energy loss (Beskin, Gurevich & Istomin 1993; Mestel, Panagi & Shibata 1999). It is the longitudinal current closing within the pulsar polar cap that exerts the retardation torque acting on the neutron star. This torque can be determined if the structure of longitudinal current is known. Here we remind of the solution by Beskin, Gurevitch & Istomin (1993) and discuss the validity of such an assumption. The behavior of the recently observed "part-time job" pulsar B1931+24 can be naturally explained within the model of current loss while the magneto-dipole model faces difficulties.

Keywords Neutron stars · magnetosphere · pulsars

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1 Magneto-dipole loss

The first idea to explain the energy loss of radio pulsars was to consider the magneto-dipole radiation (Pacini 1967). Indeed, the magneto-dipole formula gives for the radiation power

$$W_{md} = \frac{1}{6} \frac{B_0^2 \Omega^4 R^6}{c^3} \sin^2 \chi, \quad (1)$$

where χ is the angle between rotational and magnetic axis, R is a neutron star radius ~ 10 km, and Ω is a pulsar angular velocity. This formula explains pulsar activity and observed energy loss for expected large magnetic field near the surface $B_0 \sim 10^{12}$ Gs.

Let us recall that the physical reason of such energy loss is the action of the torque exerted on the pulsar by the Ampere force of the electric currents flowing over the neutron star surface (Istomin 2005). The electric and magnetic fields in the outgoing magneto-dipole wave in vacuum can be found by solving the wave equations $\nabla^2 \mathbf{B} + \Omega^2/c^2 \mathbf{B} = 0$ and $\nabla^2 \mathbf{E} + \Omega^2/c^2 \mathbf{E} = 0$ with the boundary conditions stated as the fields corresponding components \mathbf{E}_t and \mathbf{B}_n being continuous through the neutron star surface. Inside the star one can consider magnetic field as homogeneous, and find the corresponding electric field using the frozen-in condition. As a result, the full solution will give us the discontinuity of $\{\mathbf{B}_t\}$ and $\{\mathbf{E}_n\}$ attributed to the surface charge σ_s and the surface current $\mathbf{J}_s = \frac{c}{4\pi}[\mathbf{n}, \{\mathbf{B}\}]$. The Ampere force exerts the torque

$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r}, [\mathbf{J}_s, \mathbf{B}]] dS \quad (2)$$

on the neutron star. The energy loss of a pulsar due to this torque is equal to (1). Thus, it is the surface current that is responsible for the angular momentum transform from a neutron star to an outgoing magneto-dipole wave (Michel 1991; Beskin, Gurevich & Istomin 1993).

A pulsar in vacuum loses its rotational energy due to angular momentum transform to the electromagnetic

wave at the rate given by (1). However, this is not so if the pulsar magnetosphere is filled with plasma and there is no longitudinal current in the magnetosphere. As was shown by Beskin, Gurevich & Istomin (1993) and Mestel, Panagi & Shibata (1999), in this case the Poynting flux through the light cylinder is equal to zero. Indeed, as the ideal conductivity condition is applicable not only inside the neutron star but outside as well there is no magnetic field discontinuity at the star surface. Consequently, there is no Ampere force acting on a pulsar and, hence, there is no energy loss. For zero longitudinal current the light cylinder is a natural boundary of the pulsar magnetosphere.

2 Current loss

In this section we remind the exact solution for the surface current within the polar cap presented in the monograph by Beskin, Gurevich & Istomin (1993). As it was shown in the previous section, the neutron star retardation is due to Ampere force $\mathbf{F}_A = \mathbf{J}_s \times \mathbf{B}/c$. If the magnetosphere is filled with plasma, the surface current \mathbf{J}_s is flowing within magnetic polar cap only. This surface current closes the volume longitudinal current in the magnetosphere and the return current flowing along the separatrix between open and closed field lines region.

In order to write the equation for the surface current, the several assumptions must be made. We assume that the conductivity of the pulsar surface is uniform, and the electric field \mathbf{E}_s has a potential, so that the surface current can be written as $\mathbf{J}_s = \nabla \xi'$. Using the stationary continuity equation $\text{div} \mathbf{J} = 0$, where $\partial J_z / \partial z$ is equal to the volume current $i_{||} B_0$ flowing along the open field lines, one can obtain

$$\nabla^2 \xi' = -i_{||} B_0. \quad (3)$$

Making the substitution $x = \sin \theta_m$ and introducing the non-dimensional potential $\xi = 4\pi \xi' / B_0 R^2 \Omega$ and current $i_0 = -4\pi i_{||} / \Omega R^2$ we get

$$(1 - x^2) \frac{\partial^2 \xi}{\partial x^2} + \frac{1 - 2x^2}{x} \frac{\partial \xi}{\partial x} + \frac{1}{x^2} \frac{\partial^2 \xi}{\partial \varphi_m^2} = i_0(x, \varphi_m). \quad (4)$$

Here θ_m and φ_m are polar and azimuth angles with respect to magnetic axis.

Equation (4) needs a boundary condition. This boundary condition results from the proposition that there is no surface current outside the magnetic polar cap. This means that

$$\xi[x_0(\varphi_m), \varphi_m] = \text{const}, \quad (5)$$

where $x_0(\varphi_m)$ is the polar cap boundary. The solution for the Dirichlet problem (4, 5) can be obtained by the Fourier method. The jump in the potential derivative at $x = x_0(\varphi_m)$ gives us the current flowing along the separatrix. As we see, it is defined uniquely by the longitudinal current in the region of open field lines and by

condition that no longitudinal current can flow in the region of the closed field lines.

For arbitrary inclination angle χ the current i_0 can be written as a sum of its symmetric i_S and anti-symmetric i_A components. The anti-symmetric current begins playing the main role when the pulsar polar cap crosses the surface where the Goldreich-Julian charge density $\rho_{GJ} = -\Omega \cdot \mathbf{B} / 2\pi c$ changes sign. This condition can be written as

$$\chi = \frac{\pi}{2} - \sqrt{\frac{\Omega R}{c}}. \quad (6)$$

For example, taking the Goldreich-Julian current density

$$i_{GJ}(x, \varphi_m) \approx \cos \chi + \frac{3}{2} x \cos \varphi_m \sin \chi = i_S + i_A x \cos \varphi_m, \quad (7)$$

we obtain the following solutions of the Dirichlet problem (4)-(5) for the symmetric and anti-symmetric volume currents respectively:

$$\xi_S = \frac{i_S}{4} x^2, \quad (8)$$

$$\xi_A = \frac{i_A}{8} x(x^2 - x_0^2) \cos \varphi_m. \quad (9)$$

The torque exerted by the surface current over the neuron star can be written as

$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r}, [\mathbf{J}_s, (\mathbf{B}_0)]] dS, \quad (10)$$

where \mathbf{B}_0 is the dipole field near the neutron star surface. Let us decompose the braking torque \mathbf{K} over the orthogonal system of unit vectors \mathbf{e}_m , \mathbf{n}_1 , and \mathbf{n}_2 . Here \mathbf{e}_m is a unit vector along the magnetic moment; vector \mathbf{n}_1 is perpendicular to the magnetic moment and lies in the plane of the magnetic moment and the rotational axis; vector \mathbf{n}_2 complements these to the right-hand triple:

$$\mathbf{K} = K_{||} \mathbf{e}_m + K_{\perp} \mathbf{n}_1 + K_{\dagger} \mathbf{n}_2. \quad (11)$$

K_{\dagger} plays no role in Euler equations that describe the rotational dynamics of the decelerating neutron star. As a result we have (Beskin, Gurevich & Istomin 1993):

$$K_{||} = -\frac{B_0^2 \Omega R^4}{2\pi c} \int_0^{2\pi} d\varphi_m \int_0^{x_0(\varphi_m)} dx x^2 \sqrt{1 - x^2} \frac{\partial \xi}{\partial \varphi_m}, \quad (12)$$

$$K_{\perp} = K_1 + K_2, \quad (13)$$

$$K_1 = \frac{B_0^2 \Omega R^4}{2\pi c} \int_0^{2\pi} d\varphi_m \int_0^{x_0(\varphi_m)} dx A, \quad (14)$$

$$K_2 = \frac{B_0^2 \Omega R^4}{2\pi c} \int_0^{2\pi} d\varphi_m \int_0^{x_0(\varphi_m)} dx x^3 \cos \varphi_m \frac{\partial \xi}{\partial x}, \quad (15)$$

where $A = x \cos \varphi_m \partial \xi / \partial x - \sin \varphi_m \partial \xi / \partial \varphi_m$.

The leading perpendicular torque component K_1 is equal to zero equivalently for arbitrary shape of the polar

cap due to the boundary condition (5). The values $K_{||}$ and K_{\perp} can be written as

$$K_{||} = \frac{B_0^2 \Omega^3 R^6}{c^3} \left[-c_{||} i_S + \mu_{||} \left(\frac{\Omega R}{c} \right)^{1/2} i_A \right], \quad (16)$$

$$K_{\perp} = \frac{B_0^2 \Omega^3 R^6}{c^3} \left[\mu_{\perp} \left(\frac{\Omega R}{c} \right)^{1/2} i_S + c_{\perp} \left(\frac{\Omega R}{c} \right) i_A \right]. \quad (17)$$

Here the coefficients $\mu_{||}$ and μ_{\perp} depending on the shape of the polar cap are much less than unity, and the coefficients $c_{||}$ and $c_{\perp} \sim 1$.

We can now find the derivatives of the angular velocity $\dot{\Omega}$ and of the inclination angle $\dot{\chi}$ of a neutron star through the Euler dynamics equations:

$$J_r \frac{d\Omega}{dt} = K_{||} \cos \chi + K_{\perp} \sin \chi, \quad (18)$$

$$J_r \Omega \frac{d\chi}{dt} = K_{\perp} \cos \chi - K_{||} \sin \chi. \quad (19)$$

For the inclination angles χ not too close to 90° (i.e., for $\cos \chi > (\Omega R/c)^{1/2}$), when the anti-symmetric current plays no role in the neutron star dynamics, we obtain

$$\frac{d\Omega}{dt} = -c_{||} \frac{B_0^2 \Omega^3 R^6}{J_r c^3} i_S \cos \chi, \quad (20)$$

$$\frac{d\chi}{dt} = c_{||} \frac{B_0^2 \Omega^3 R^6}{J_r c^3} i_S \sin \chi. \quad (21)$$

As a result, for homogeneous current density within open magnetic field lines region $i_S = j_{||}/j_{GJ} = \text{const}$ where $j_{GJ} = c\rho_{GJ}$ we have

$$W_c = \frac{f_*^2}{4} \frac{B_0^2 \Omega^4 R^6}{c^3} i_S \cos \chi. \quad (22)$$

Here f_* is the non-dimensional area of a pulsar polar cap: $S_{\text{cap}} = f_* \pi (\Omega R/c)$. It depends on the structure of the magnetic field near the light cylinder. For a pure dipole magnetic field (and aligned rotator) $f_* = 1$, and for a magnetosphere containing no longitudinal currents f_* changes from 1.592 for the aligned rotator, $\chi = 0^\circ$ (Michel 1991), to 1.96 for an orthogonal rotator, $\chi = 90^\circ$ (Beskin, Gurevich & Istomin 1993). Recent numerical calculations for an axisymmetric magnetosphere with non-zero longitudinal electric current give $f_* \approx 1.23 - 1.27$ (Gruzinov 2005; Komissarov 2006; Timokhin 2006). If the singular point separating open and close field lines can be located inside the light cylinder, the value f_* can be even $\gg 1$. As the Goldreich-Julian charge density near the polar cap is proportional to $\cos \chi$, one can write

$$W_c \approx \frac{f_*^2}{4} \frac{B_0^2 \Omega^4 R^6}{c^3} \cos^2 \chi. \quad (23)$$

On the other hand for $\chi \approx 90^\circ$ when the anti-symmetric current plays the leading role we obtain

$$W_c \approx \frac{B_0^2 \Omega^5 R^7}{c^4}. \quad (24)$$

As we see, the energy loss of the orthogonal rotator are $\Omega R/c$ times smaller than of the aligned rotator.

3 PSR B1931+24

The recent discovery of the "part-time job" pulsars PSR B1931+24 (Kramer et al. 2006) with $\dot{\Omega}_{\text{on}}/\dot{\Omega}_{\text{off}} \approx 1.5$ shows that the current loss is indeed playing an important role in the pulsar energy loss. If we assume that in the on-state the energy loss is connected with the current loss only and in the off-state with the magneto-dipole radiation (in which case the magnetosphere must be not filled with plasma) we get

$$\frac{\dot{\Omega}_{\text{on}}}{\dot{\Omega}_{\text{off}}} = \frac{3f_*^2}{2} \cot^2 \chi. \quad (25)$$

It gives $\chi \approx 60^\circ$. On the other hand, if we assume the Spitkovsky's relation for the on-state energy loss (Spitkovsky 2006)

$$W_{\text{tot}} = \frac{1}{4} \frac{B_0^2 \Omega^4 R^6}{c^3} (1 + \sin^2 \chi), \quad (26)$$

we obtain

$$\frac{\dot{\Omega}_{\text{on}}}{\dot{\Omega}_{\text{off}}} = \frac{3}{2} \frac{(1 + \sin^2 \chi)}{\sin^2 \chi}. \quad (27)$$

Clear, this ratio cannot be equal to 1.5 for any inclination angle. This discrepancy can be connected with that fact that all the numerical calculations produced recently contain no restriction on the longitudinal electric current magnitude. As a result, current density can be much larger than Goldreich-Julian one.

4 On the magnitude of a surface current

As we have shown, the current loss plays the major role in the pulsar dynamics. In particular, the behaviour of the pulsar B1931+24 can be naturally explained within this model. The current loss model have some important consequences:

1. the energy loss of an orthogonal rotator is $\Omega R/c$ times smaller than for an aligned rotator. This is connected with the boundary condition (5) that leads to almost full screening of the toroidal magnetic field in the open field lines region (see Beskin & Nokhrina (2004));
2. consequently, during its evolution a pulsar inclination angle tends to $\pi/2$ where energy loss is minimal.

On the other hand, it is known for the Michel's monopole solution that in order to have the MHD flow up to infinity, the toroidal magnetic field must be of the same order as the poloidal electric field on the light cylinder. If the longitudinal current $j_{||}$ does not exceed by $(\Omega R/c)^{-1/2}$ times the respective Goldreich-Julian current density (for the typical pulsars this factor approach the value of 10^2), the light surface $|\mathbf{E}| = |\mathbf{B}|$ for the orthogonal rotator must be located in the vicinity of the light cylinder. In this case the effective energy conversion

and the current closure is to take place in the boundary layer near the light surface (Beskin, Gurevich & Istomin 1993; Chiueh, Li & Begelman 1998; Beskin & Rafikov 2000).

In order for these results being not true (for example, in order for the light surface being removed to infinity) there must be a sufficient change in the current density value in the inner gap. We should emphasize that for the model with free particle escape it is hard to support the current different than the Goldreich-Julian current. Indeed, since ρ_{GJ} is the particle density needed to screen the longitudinal electric field, the value for the current must be close $j_{GJ} = c\rho_{GJ}$. In order to change this value significantly we must support the plasma inflow in the inner gap region (Lyubarskii 1992). For example, these particles can be produced in the outer gap. But for different poloidal field configuration it is obvious that the major number of field lines intersect the outer gap region outside the Alfvénic surface: as it was shown for several field configurations by Beskin, Kuznetsova & Rafikov (1998) and Beskin & Nokhrina (2006), inside the fast magnetosonic surface the flow remains still highly magnetized. Thus, the deviation of current lines from the field lines is negligible. On the other hand, magnetized plasma can intersect the Alfvénic surface outwards only (see Beskin (2006) for more detail). Thus, the outer gap can not significantly affect the current in the vicinity of the polar cap.

Finally, it is necessary to stress that the recent numerical calculations by Gruzinov (2005), Komissarov (2006), Timokhin (2006), Spitkovsky (2006) do not include into consideration the condition that the longitudinal current density must be close to j_{GJ} . In all these works the authors assume that there may be any current flowing through the cascade region. However, if this is not so, and the current is indeed close to the Goldreich-Julian current, the structure of a magnetosphere may be different from the one obtained in the numerical simulations.

density in the open field line region is to be much larger than j_{GJ} .

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5 Conclusions

As we have seen, the current loss connected with the longitudinal current flowing in the magnetosphere plays the main role in pulsar dynamics, and recent observations of “part-time job” pulsar supports this point. This evolution includes not only a neutron star retardation but also the sufficient change in the angle between magnetic and spin axis. We have seen as well that the model of current loss depends crucially on the distribution of the electric current and its value in the inner gap. For current loss model with j_{GJ} the inclination angle grows with time so a pulsar tends to be an orthogonal rotator. In this case the energy loss is to be $\Omega R/c$ times smaller than for the aligned rotator. As a consequence, the light surface must be located in the very vicinity of the light cylinder. On the other hand, to realize the homogeneous MHD outflow up to infinity for the orthogonal rotator the current